## Quiz 22: Continuous Probability II

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This quiz does not count towards your grade. It exists to simply gauge your understanding. Treat this as though it were a portion of your midterm or final exam.

## 1 More Lightbulbs

Consider $n$ lightbulbs, where the lifetime of each is exponentially distributed with parameter $\lambda$.

Hint: Let $X_{i}$ denote the lifetime of the $i$ th lightbulb. We know that $X_{i} \sim \operatorname{Expo}(\lambda)$.

1. Compute the PDF of $Z=X_{1}+X_{2}$. We are re-deriving the Erlang distribution.
2. Compute the PDF of $Y=X_{1}+X_{2}+X_{3}$. Do you notice a pattern?
3. Take $N \sim \operatorname{GEOM}(p)$ for some constant $p$. Compute the PDF of $X=\sum_{i=1}^{N} X_{i}$. What do you observe? Directly apply the PDF of the Erlang distribution.

$$
\frac{\lambda^{k} x^{k-1} e^{-\lambda x}}{(k-1)!}
$$

