Quiz 14 : Geometric, Poisson Solutions

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This quiz does not count towards your grade. It exists to simply gauge your understanding. Treat this as though it were a portion of your midterm or final exam.

1 Distributions

Alvin has trouble typing his 9-character alphanumeric password into his computer correctly. Assume that he types all 9 characters before hitting enter and typing a single character is independent of typing any other character.

1. If Alvin takes on average b times to enter his password in, what is the probability p that he hits a key incorrectly?

   Solution: Assume we have $X \sim \text{GEOM}(p)$, the number of times it takes to enter a password fully correctly, where q is the probability that he types in a password fully-correctly. We know that $E[X] = \frac{1}{q} = b$. Let p be the probability that the ith character is typed incorrectly, then q is $(1-p)^9$. Finally, we have that

   $$\begin{aligned} \frac{1}{q} &= b \\ (1-p)^{-9} &= b \\ b^{-1} &= (1-p)^9 \\ b^{-1/9} &= 1-p \\ p &= 1-b^{-1/9} \end{aligned}$$

2. Alvin has mis-typed his password t times. What is the probability that Alvin takes v more tries before he can unlock his computer? Express in terms of v and b.

   Solution: This is an application of the memoryless property, as we're looking at

   $$\Pr(X > v+t | X > t) = \Pr(X > v)$$

Again, let q be the probability that he types in a password fully-correctly. So, the probability that he tries at least v more times, is the probability that he fails all v times. Thus,

$$\Pr(X > v) = (1-q)^v$$

We know that $q = \frac{1}{b}$, so plug in to obtain our final expression.

$$\Pr(X > v) = (1-b^{-1})^v$$

3. If Alvin writes an essay with dozens of pages, where each word is exactly 9 letters, with what probability will he mis-type none of his words? Let there be 500 words per page, and express in terms of b.

   Solution: Consider W, the number of mis-typed words. We know from part a that q is the probability of typing a word correctly. This is the Poisson distribution, distributed with parameter $500(1-q) = 500(1-\frac{1}{b})$. Let $W \sim \text{Poiss}(500(1-\frac{1}{b})$ denote the number of words mis-typed. We compute $\Pr[W = 0]$, by plugging into the formula.

$$\Pr[W = 0] = e^{-500(1-\frac{1}{b})}$$

4. Considering all possible values of b, is it possible for him to type a perfect essay with complete certainty? (probability of 0 typos with 100% certainty) Assume that Alvin *never* types in his password correctly on the first try.

   Solution: No. :(

   The only way for $\Pr[W = 0] = e^{-500(1-\frac{1}{b})}$ to be 1, is if $1 - \frac{1}{b} = 0$ implying $b = 1$. This means Alvin needs only one try to type in his password correctly, in expectation. This is impossible, by hypothesis above.