Crib 22 : Continuous Probability II

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The crib sheet contains cheat-sheet worthy information but is not a substitute for lectures or for reading the notes. It also contains pointers and common mistakes.

1 Terminology, Notation

- The **probability density function (PDF)** of a continuous random variable X is $f_X(x)$. Remember to specify valid values for X.
- The **probability mass function (PMF)** of a discrete random variable X is specified for all valid values of X. (e.g., $\Pr(X = x_1) = p_1, \Pr(X = x_2) = p_2 \dots$)
- The **cumulative density function (CDF)** of a continuous random variable X is $\Pr(X \leq x) = \int_{-\infty}^{x} f_X(t)dt$.
- The CDF of a discrete random variable X is $\Pr(X \leq x) = \sum_x \Pr(X = x)$

2 Computation

- Computing PDF (Method 1): Take the CDF of your random variable, and then differentiate to get the PDF.
- Computing PDF (Method 2): For a sum of *independent* random variables, e.g., $Z = X + Y$, we can use the convolution formula. (See 4.1 for proof)

$$f_Z(z) = \int_{-\infty}^{\infty} f_X(z-y)f_Y(y)dy$$

3 Erlang Distribution

This distribution is technically out-of-scope for this course. However, it helps us understand analogies between continuous and discrete distributions.

- The sum of a fixed k number of exponentials is distributed according to the **Erlang Distribution**.
- The limiting distribution for the binomial distribution is the Poisson.
- The limiting distribution for the geometric distribution is the exponential.
- A sum of geometric distributions is known as the negative binomial distribution.
- A sum of exponential distributions is known as the Erlang distribution.
- The sum of a geometric number of exponentials is exponential. Formally, for $N \sim \textsc{Geom}(p)$ and i.i.d. $X_i \sim \textsc{Expo}(\lambda)$, $X = \sum_{i=1}^{N} X_i \sim \textsc{Expo}(\lambda p)$.