Crib 11

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The crib sheet contains cheat-sheet worthy information but is not a substitute for lectures or for reading the notes. It also contains pointers and common mistakes.

1 Counting

- If we have k items, there are $k!$ ways to order them.
- If we have k items, each with n options, then we have n^k total combinations.
- If we have k items, each with n_i options, then we have $n_1 \cdot n_2 \cdots n_k$ total combinations.
- If we are counting the total number of anagrams of a given word w: compute a, the total number of letters in w, and for each repeated letter, compute the number of times it repeats r_i. The number of possible anagrams is thus the following, where k is the number of distinct letters that repeat.

$$\frac{w!}{r_1!r_2!...r_k!}$$

For example, consider the number of anagrams of "SENPAISINHOCHEWI". The total number of letters is 17. There are 2 Ss, 2 Es, 2 Ns, and 3 Is. Thus, we have $\frac{17!}{2!2!2!3!}$ *anagrams.*

- If we are splitting k *indistinguishable* items among n slots, we use stars and bars.
- The inclusion-exclusion principle says that the union of two sets

  $$A \cup B = A + B - A \cap B$$

  Intuitively, think of Venn Diagrams. We can add two circles, then subtract the overlap once to get the full Venn Diagram. Likewise, the inclusion-principle also states that

  $$A \cup B \cup C = A + B + C - (A \cap B) - (B \cap C) - (C \cap A) + (A \cap B \cap C)$$

  Again, add all three circles, subtract pairwise overlap, and finally add the centerpiece.